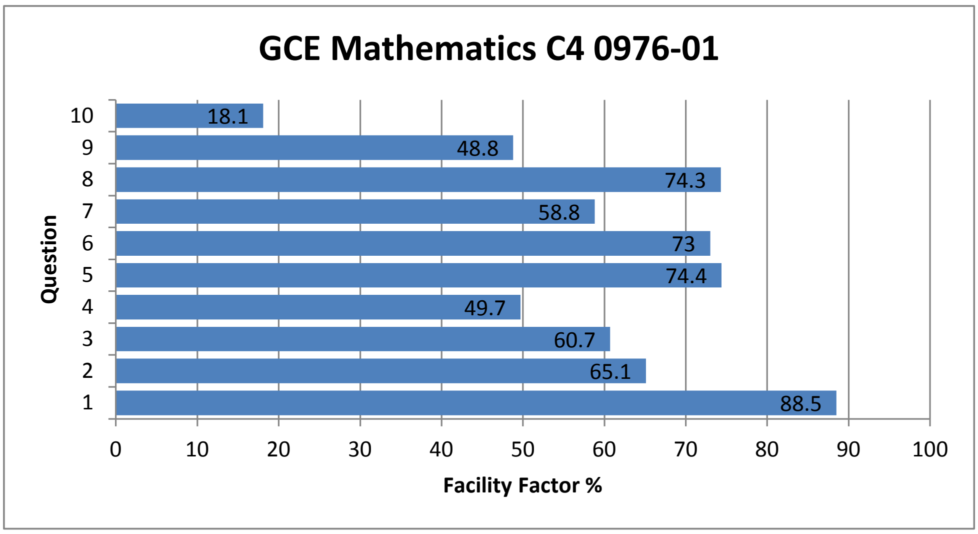
WJEC 2014 Online Exam Review

GCE Mathematics C4 0976-01

All Candidates' performance across questions

Question Title	*N*	*Mean*	*S D*	*Max Mark*	*F F*	*Attempt %*	
1	3157	4.4	1	5	88.5	99.8	
2	3157	3.9	0.9	6	65.1	99.8	
3	3137	6.1	2.7	10	60.7	99.2	
4	3106	3	2.1	6	49.7	98.2	4
5	3080	5.2	2	7	74.4	97.3	
6	3132	6.6	1.9	9	73	99	6
7	3109	4.7	2.6	8	58.8	98.3	
8	3066	7.4	3	10	74.3	96.9	
9	3066	5.4	3	11	48.8	96.9	
10	2880	0.5	1	3	18.1	91	10



4. The region R is bounded by the curve $y = 3 + 2\sin x$, the x-axis and the lines $x = 0, x = \dfrac{\pi}{4}$.

Find the volume of the solid generated when R is rotated through four right angles about the x-axis. Give your answer correct to the nearest integer. [6]

(4.) $y = 3 + 2\sin x$

$y^2 = (3 + 2\sin x)^2 \cancel{= 9 + 12\sin x + 4\sin^2 x}$

$$V = \pi \int_0^{\pi/4} y^2 \, dx = \pi \int_0^{\pi/4} (3 + 2\sin x)^2 \, dx$$

$$= \pi \left[\frac{(3 + 2\sin x)^3}{3 \times 2\cos x} \right]_0^{\pi/4}$$

$$= \pi \left[\frac{3 + 2\sin x}{6\cos x} \right]_0^{\pi/4}$$

$$= \pi \left[\frac{3 + \sqrt{2}}{3\sqrt{2}} - \frac{3}{6} \right]$$

$$= \pi \left[\frac{-1 + 3\sqrt{2}}{6} \right]$$

$$= 2$$

4.

$y = 3 + 2\sin x$

$y^2 = (3+2\sin x)^2 \cancel{= 9 + 12\sin x + 4\sin^2 x}$

$V = \pi \int_0^{\pi/4} y^2\, dx = \pi \int_0^{\pi/4} (3+2\sin x)^2\, dx$

$= \pi \left[\frac{(3+2\sin x)^3}{3 \times 2\cos x} \right]_0^{\pi/4}$

$= \frac{\pi}{\cancel{\;}} \left[\frac{3+2\sin x}{6\cos x} \right]_0^{\pi/4}$

$= \pi \left[\frac{3+\sqrt{2}}{3\sqrt{2}} - \frac{3}{6} \right]$

$= \pi \left[\frac{-1+3\sqrt{2}}{6} \right]$

$= 2$

B1

M1

M1

B1

M0

A0

4. $y = 3 + 2\sin x \qquad x = 0 \quad x = \pi/4$

$$V = \int \pi y^2 \, dx$$

$$\int_0^{\pi/4} \pi \times (3 + 2\sin x)^2 \, dx$$

$$\pi \int_0^{\pi/4} (3 + 2\sin x)(3 + 2\sin x) \, dx$$

$$\pi \int_0^{\pi/4} 9 + 12\sin x + 4\sin^2 x \quad dx$$

$$\pi \Big[9x + 12\cos x + 4\cos^2 x \Big]_0^{\pi/4}$$

$= 4.88$

$= 5.$

4. $y = 3 + 2\sin x$ $\quad x = 0 \quad x = \pi/4$

$$V = \int \pi y^2 \, dx$$

$$\int_0^{\pi/4} \pi \times (3 + 2\sin x)^2 \, dx$$ B1

$$\pi \int_0^{\pi/4} (3 + 2\sin x)(3 + 2\sin x) \, dx$$

$$\pi \int_0^{\pi/4} 9 + 12\sin x + 4\sin^2 x \quad dx$$

$$\pi \Big[9x + 12\cos x + 4\cos^2 x \Big]_0^{\pi/4}$$ M0

$= 4.88$

$= 5.$ A0

4. $y = 3 + 2\sin x$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 4x = 1 - 4\sin^2 x$

$4\sin^2 x = 1 - \cos 4x$

$$\pi \int_0^{\frac{\pi}{4}} (3 + 2\sin x)^2 \, dx$$

$$\pi \int_0^{\frac{\pi}{4}} 9 + 4\sin^2 x \, dx$$

$$\pi \int_0^{\frac{\pi}{4}} 9 + 1 - \cos 4x \, dx$$

$$\pi \left[9x + x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{4}}$$

$$\pi \left[10x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{4}}$$

$$\pi \left[\left(\frac{5\pi}{2} - 0 \right) - \left(0 - 0 \right) \right]$$

$$= \frac{5\pi^2}{2}$$

4. $y = 3 + 2\sin x$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 4x = 1 - 4\sin^2 x$

$4\sin^2 x = 1 - \cos 4x$

$$\pi \int_0^{\frac{\pi}{4}} (3 + 2\sin x)^2 \, dx$$ B1

$$\pi \int_0^{\frac{\pi}{4}} 9 + 4\sin^2 x \, dx$$ M1

$$\pi \int_0^{\frac{\pi}{4}} 9 + 1 - \cos 4x \, dx$$ A0

$$\pi \left[9x + x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{4}}$$ B0

$$\pi \left[10x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{4}}$$ M1

$$\pi \left[\left(\frac{5\pi}{2} - 0 \right) - \left(0 - 0 \right) \right]$$

$$= \frac{5\pi^2}{2}$$ A0

6. The curve C has the parametric equations $x = 2t$, $y = 5t^3$. The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$2y = 15p^2x - 20p^3.$$

[4]

(b) The tangent to C at the point P intersects C again at the point $Q(2q, 5q^3)$. Given that $p = 1$, show that q satisfies the equation

$$q^3 - 3q + 2 = 0.$$

Hence find the value of q. [5]

6a) $x=2t$, $y=5t^3$

$\frac{dx}{dt}=2$ $\qquad \frac{dy}{dt}=15t^2$

$\frac{dy}{dx}=\frac{15t^2}{2}$

$\frac{y-5p^3}{x-2p}=\frac{15p^2}{2}$

$2y-10p^3=15p^2x-30p^3$

$-20p^3+15p^2x=2y$

b) ~~Put~~ $\cancel{x=2q \quad y=5q^3}$

$\cancel{-20p^3+15p^2(2q)=2(5q^3)}$

$\cancel{-20p^3+30pq^3=10q^3}$

~~For~~ $\cancel{p=1}$

$\cancel{-20+30q^3-20p^3=0}$

b) Put $x=2q$ $y=5q^3$

$2(5q^3)=15p^2(2q)-20p^3$

$10q^3=30p^2q-20p^3$

For $p=1$

$10q^3=30q-20$

$q^3-3q+2=0$

$(q-1)(q^2-2)=0$

$q=1$ or $q=\pm\sqrt{2}$

6a) $x=2t$, $y=5t^3$

$\frac{dx}{dt}=2$ $\frac{dy}{dt}=15t^2$

$\frac{dy}{dx}=\frac{15t^2}{2}$

$\frac{y-5p^3}{x-2p}=\frac{15p^2}{2}$

$2y-10p^3=15p^2x-30p^3$

$-20p^3+15p^2x=2y$

b) ~~Put $x=2q$ $y=5q^3$~~

~~$-20p^3+15p^2(2q)=2(5q^3)$~~

~~$-20p^3+30pq=10q^3$~~

~~For $p=1$~~

~~$-20+30q^3-10q^3=0$~~

b) Put $x=2q$ $y=5q^3$

$2(5q^3)=15p^2(2q)-20p^3$

$10q^3=30p^2q-20p^3$

For $p=1$

$10q^3=30q-20$

$q^3-3q+2=0$

$(q-1)(q^2-2)=0$

$q=1$ or $q=\pm\sqrt{2}$

6b) given that $p=1$, $2y = 15x - 20$

@ $Q(2q, 5q^3)$, $2(5q^3) = 15(2q) - 20$

$$10q^3 = 30q - 20$$

$$q^3 = 3q - 2$$

$$\therefore q^3 - 3q + 2 = 0$$

$$q^3 - 3q + 2 = 0$$

$$q^3 - 3q = -2$$

$$q(q^2 - 3) = -2$$

either $q = -2$ or $q^2 = 1$

$$q = 1$$

6b) given that $p=1$, $2y = 15x - 20$

@ $Q(2q, 5q^3)$, $2(5q^3) = 15(2q) - 20$

$10q^3 = 30q - 20$

$q^3 = 3q - 2$

$\therefore q^3 - 3q + 2 = 0$

M1

A1

$q^3 - 3q + 2 = 0$

$q^3 - 3q = -2$

$q(q^2 - 3) = -2$

either $q = -2$ or $q^2 = 1$

$q = 1$

M0

A0

6. $x = 2t \qquad y = 5t^3$

$\frac{dx}{dt} = 2. \qquad \frac{dy}{dt} = 15t^2.$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{15t^2}{2}$ is the gradient of the tangent.

x @ $P = 2p$

y @ $P = 5p^3$.

equation:

$$y - 5p^3 = \frac{15p^2}{2}(x - 2p).$$

$$2y - 10p^3 = 15p^2(x - 2p).$$

$$2y - 10p^3 = 15p^2x - 30p^3.$$

$$\underline{\underline{2y = 15p^2x - 20p^3}}$$

b. when $p = 1$

$$2y = 15x - 20$$

$$2(5q^3) = 15(2q) - 20$$

$$10q^3 = 30q - 20 \qquad \div 10$$

$$q^3 = 3q - 2.$$

$$\underline{\underline{q^3 - 3q + 2 = 0}}$$

to find q complete the square.

$$q\left(q - \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = 0$$

$$q\left(q - \frac{3}{2}\right)^2 = \frac{1}{4}.$$

$q = \underline{\frac{1}{4}}$ or $q - \frac{3}{2} = \frac{1}{2}.$

$$q = \underline{\underline{2}}$$

6. $x = 2t \qquad y = 5t^3$

$$\frac{dx}{dt} = 2. \qquad \frac{dy}{dt} = 15t^2.$$ M1

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{15t^2}{2.}$$ is the gradient of the tangent. A1

x @ P $= 2p$

y @ P $= 5p^3$.

equation:

$$y - 5p^3 = \frac{15p^2}{2}(x - 2p).$$ M1

$$2y - 10p^3 = 15p^2(x - 2p).$$

$$2y - 10p^3 = 15p^2x - 30p^3.$$

$$\underline{\underline{2y = 15p^2x - 20p^3}}$$ A1

b. when $p = 1$

$$2y = 15x - 20$$

$$2(5q^3) = 15(2q) - 20$$ M1

$$10q^3 = 30q - 20 \qquad \div 10$$

$$q^3 = 3q - 2.$$

$$\underline{\underline{q^3 - 3q + 2 = 0}}$$ A1

to find q complete the square.

$$q\left(q - \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = 0$$ M0

$$q\left(q - \frac{3}{2}\right)^2 = \frac{1}{4}.$$

$q = \underline{\frac{1}{4}}$ or $q - \frac{3}{2} = \frac{1}{2}.$

$q = \underline{2}$

10. Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

for all values of θ.

Assume that there is a value of θ for which $sin\theta + cos\theta > \sqrt{2}$.
Then squaring both sides, we have:

[3]

END OF PAPER

10. Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$

Then, squaring both sides we get.

$$\sin^2\theta + \cos^2\theta > \sqrt{2}$$

But $\sin^2\theta + \cos^2\theta \equiv 1$

Thus $1 > \sqrt{2}$

As $\sqrt{2} \cong 1.41$, it cannot be less than 1

This is a contradiction, so $\sin\theta + \cos\theta \leq \sqrt{2}$

10. Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$
Then, squaring both sides we get.

$$\sin^2\theta + \cos^2\theta > \sqrt{2}$$

But $\sin^2\theta + \cos^2\theta = 1$

Thus $1 > \sqrt{2}$

As $\sqrt{2} \approx 1.41$, it cannot be less than 1
This is a contradiction, so $\sin\theta + \cos\theta \leq \sqrt{2}$

B0

B0

B0

(b) Assume there is a value of θ
for which $\sin\theta + \cos\theta > \sqrt{2}$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta > 2$$

$$1 - \cos^2\theta + 2\cos\theta\sin\theta + \cos^2\theta > 2$$

$$2\cos\theta\sin\theta > 1$$

$$\cos\theta\sin\theta > 1$$

Max. value of $\cos\theta = 1$
Max value of $\sin\theta = 1$
~~$\cos\sin\theta$~~
$\therefore$ Max value of $\cos\theta\sin\theta = 1$,
which is not greater than 1
there is a contradiction
The assumption is false
and $\sin\theta + \cos\theta \leq \sqrt{2}$

(b)	Assume there is a value of θ
	for which $\sin\theta + \cos\theta > \sqrt{2}$
	$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta > 2$
	$1 - \cos^2\theta + 2\cos\theta\sin\theta + \cos^2\theta > 2$
	$2\cos\theta\sin\theta > 1$
	~~$\cos\theta \sin\theta > 1$~~
	Max. value of $\cos\theta = 1$
	Max value of $\sin\theta = 1$
	~~$\cos\theta$~~
	$\therefore$ Max value of $\cos\theta\sin\theta = 1$
	which is not greater than 1
	there is a contradiction
	The assumption is false
	and $\sin\theta + \cos\theta \leq \sqrt{2}$

10. Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$ Then squaring both sides we have

$$(\sin\theta + \cos\theta)^2 > (\sqrt{2})^2$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta > 2$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 2 > 0$$

$$2\sin\theta\cos\theta + 1 - 2 > 0$$

$$2\sin\theta\cos\theta - 1 > 0$$

$$\cancel{2\sin\theta\, 2\cos\theta}$$

$$2\sin\theta\cos\theta > 1$$

$$\cos\theta\,(2\sin\theta) > 1$$

$$\cos\theta > 1 \qquad 2\sin\theta > 1$$

$$\sin\theta > 1/2$$

However $\cos\theta \not> 1$ $\therefore$ contradiction as $\cos\theta$ cannot have values greater than 1 $\therefore$ the assumption is false and

$$\sin\theta + \cos\theta \leq \sqrt{2}$$

10. Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$ Then squaring both sides we have.

$$(\sin\theta + \cos\theta)^2 > (\sqrt{2})^2$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta > 2$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 2 > 0$$

$$2\sin\theta\cos\theta + 1 - 2 > 0$$

$$2\sin\theta\cos\theta - 1 > 0$$

$$\cancel{2\sin\theta \geq \cos\theta}$$

$$2\sin\theta\cos\theta > 1$$

$$\cos\theta\,(2\sin\theta) > 1$$

$$\cos\theta > 1 \qquad 2\sin\theta > 1$$

$$\cancel{2}\sin\theta > 1/2$$

However $\cos\theta \ngtr 1$ $\therefore$ contradiction as $\cos\theta$ cannot have values greater than 1 $\therefore$ the assumption is false and

$$\sin\theta + \cos\theta \leq \sqrt{2}$$